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BEHAVIOR OF CABLE TRUSSES UNDER IMPULSE LOADS

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ABSTRACT

Cable roofs are one of the few possible solutions to cover large column free area. Being flexible in nature, these exhibit large deformations under both static and dynamic loads. It is therefore, important to consider their behavior under various types of loading before deciding a form to cover a proposed area. A study has been carried out to understand the behavior of a typical type of cable truss under dynamic loads. Response of the truss under various types of impulse loads is reported herein.

Keywords: flexible roofs, cable trusses, impulsive loads, dynamic response, integration method, non-linearity

1. INTRODUCTION

1.1 Truss forms

Cable trusses are one of three types of cable suspended roofs. Other two types being freely suspended cable roofs and cable net roofs, Figure 1. The use of freely suspended roofs is limited to small spans because of stability problems unless they are tensioned through pre-loading. Pre-tensioned cable nets and plane trusses in various forms have been used successfully for large spans. Amongst all these, the simplest form is the truss which consists of two curved cables of opposite curvature connected by vertical or inclined ties or struts, also called hangers, Figure 2

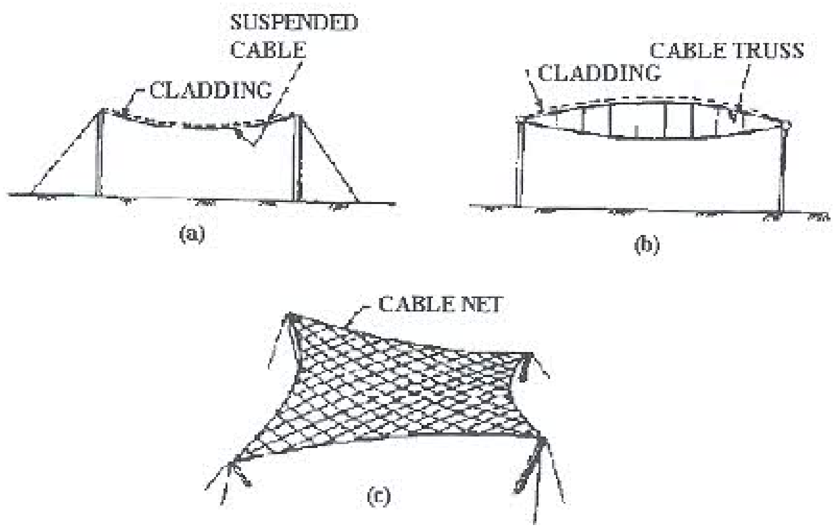

Figure 1 Cable roofs.

Cable trusses can be employed to cover rectangular, circular, elliptical or other plan areas quite conveniently.

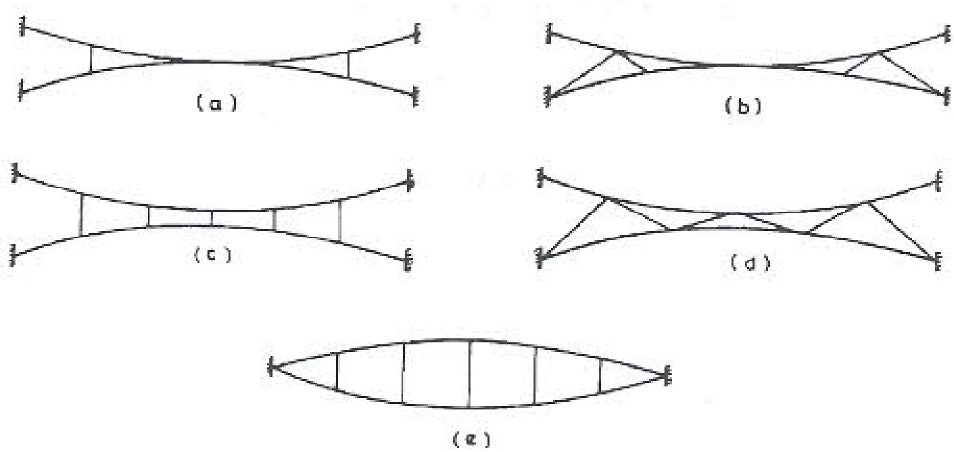

Figure 2 Cable truss forms.

1.2 Continuum and discrete element system

Analysis of a cable roof system can be done treating it as a continuum or a discrete element system. Membrane analysis is made treating the system as a small-deflection problem to arrive at approximate closed-form solutions. Use of the discrete system approach for analysis is more versatile. Masses are assumed to be lumped at nodes in this method. In the present study, the cable truss has been considered as discrete element system.

1.3 Datum for dynamic response

Two approaches are possible to select datum for obtaining response of cable roof systems under dynamic loads. While the equilibrium position at deal load is generally used as datum, some authors suggest the use of the equilibrium position at dead load plus half the live load [1] Kundson [2] obtained the quasi-static equilibrium configuration of the Aden Airways Network defined by the mean wind using a non-linear iterative procedure. Further, it was assumed that linear oscillations take place about that configuration due to fluctuations in wind. While evaluating dynamic response of the cable truss in the present study, the equilibrium position at dead load has been taken as the datum.

1.4 Solution techniques

The solution of the dynamic equilibrium equation of a multi-degree freedom system can be obtained by using the Model superposition method or the direct integration methods. Whereas the former has commonly been used for linear dynamic response analysis of flexible structures [2-5] the latter are being used to tackle non-linear problems [6]. However, certain publication [7] indicates that the Model superposition technique could also be used for the non-linear dynamic response of flexible structures. In the present study the direct integration methods have been employed to solve the dynamic equilibrium equation.

1.5 Available information

The influence of various parameters on the dynamic response of cable networks has been the subject of few studies [2-4, 7-10]. Such information on cable trusses, however, seems to be non-existent except for few studies [3,9,11].

2. METHOD OF ANALYSIS

2.1 Static response analysis

As mentioned above, the equilibrium position of the truss at dead load had been taken as the datum for evaluating dynamic response of the truss. Pre-tensioned cable systems can be analysed for static loads by treating them as discrete element systems using one of the two methods, namely (i) stiffness (displacement) method and (ii) flexibility (force) method. Though some authors have developed analytical procedures using flexibility approach, yet this approach has not found such extensive application as the stiffness method. Sometimes mixed approach has also been used but it does not offer much advantage [1]. Both these methods will result in non-linear equations, which are to be solved by suitable numerical techniques.

A tension-coefficient approach, leading to a stiffness matrix solution, has found wide coverage and application in recent years due to its simplicity and versatility and thus has been adopted in the present study. In this approach, the equation of static equilibrium for a cable structure can be expressed in matrix form as

$$\mathbf{KU} = -\mathbf{P} + \mathbf{R} \tag{1}$$

In which **K** is the square stiffness matrix consisting of coefficients of unknown joint displacements u, v and w along x, y and z axes. U is the vector of joint displacements u, v and w, **P** is the load vector and **R** is the vector of quadratic and cubic functions of joint displacements. Direct solution of Eq. 1, to obtain the nodal displacements along x, y and z axes, is not possible because the right-hand side contains vector **R**, which is a function of unknown joint displacements. It is, therefore, to be solved using a suitable numerical iterative scheme. Amongst the various available schemes, the Newton-Raphson method, being one of the effective schemes, has been employed for the static response analysis of cable truss under dead loads.

2.2 Dynamic response analysis

2.2.1 Equation of motion

Dynamic equilibrium equation or the equation of motion for a lumped-mass multi-degree freedom, non-linear system can be written in matrix form as

$$\mathbf{M\ddot{U}} + \mathbf{C\dot{U}} + \mathbf{KU} + \mathbf{R} = \mathbf{F(t)} \tag{2}$$

in which **M**, **C** and **K** are the mass, damping and linear stiffness matrices respectively. **R** is the vector of quadratic and cubic functions of joint displacements and **F(t)** is the time dependent external load vector. U, $\dot{U}$ and $\ddot{U}$ are the displacement, velocity and acceleration vectors respectively. If non-linear terms are dropped, Eq. 2 reduces to

$$M\ddot{U} + C\dot{U} + KU = F(t) \tag{3}$$

2.2.2 Stiffness matrix

The dynamic response of the system can be obtained by solving Eq. 3 using a suitable technique. If a step-by-step numerical procedure is adopted, the stiffness matrix **K** can be evaluated afresh in every time step to account for the non-linearity. To begin with, the coefficients of linear stiffness matrix **K** are calculated based upon the geometry and member forces corresponding to the static equilibrium position. The deflections, velocities and accelerations are then calculated at time $t=\Delta t$ is the time step of integration. Then using new values of displacements and member forces, the matrix **K** is recalculated and the response evaluated again for $t=\Delta t$. This step is repeated till the desired accuracy is achieved in the values of displacements. Then using the latest available matrix **K** for $t = \Delta t$, the response is calculated for $t = 2\Delta t$ and so on. In this manner, the effect of non-linearity is accounted for in stiffness matrix at every time step.

2.2.3 Damping matrix

Damping plays a very important role in dynamic response analysis but an accurate assessment of its value is difficult. In the present study damping has been neglected for obtaining response of the cable truss under impulse loads.

2.2.4 Integration method

There are a number of direct integration methods which can be employed for the solution of Eq. 3, like Central Difference Method, Linear Acceleration Method, Wilson-υ Method, Newmark Method, Houbolt Method, Fu Method, Trujillo's Method etc. [12]. Whereas some of these methods are conditionally stable, others are unconditionally stable. In the present study, Wilson-υ Method and Newmark Method with $\beta=1/4$ are employed because firstly both of them are unconditionally stable and secondly they give results close to the exact solution.

2.2.5 Time step

The time step of integration in case of conditionally stable methods is chosen such that the time step is short enough to (i) ensure convergence, (ii) ensure correct representation of the loading history, and (iii) ensure correctness of the results obtained. In case of unconditionally stable methods only the last two points are important. In the study reported herein time step of integration is taken as 0.02 sec. Irrespective of the natural period of the system. However, while studying the effect of time step on the response, three values of time step namely 0.005, 0.010 and 0.020 sec. have been used.

3. THE RESPONSE STUDY

3.1 Cable truss

The cable truss studied is shown in Figure 3. It consists of one main cable, one tie-down cable and five hangers. The span of the truss is 60 m and dip of main and tie-down cables are 4 m each. Cross sectional areas of main cable, tie-down cable and hangers are 2000 mm^2, 1300 mm^2 and 150 mm^2 respectively. Modulus of elasticity of cable is 1.5×10^5 N/mm^2 and horizontal component of pretension in main cable is taken as 600 kN. The static equilibrium position is obtained by carrying out a non-linear static analysis under the dead load of the truss acting at all the nodes and load due to membrane etc. @ 2 kN/m at the nodes on main cable only. It is assumed that due to the dynamic load the truss vibrates about the static equilibrium position.

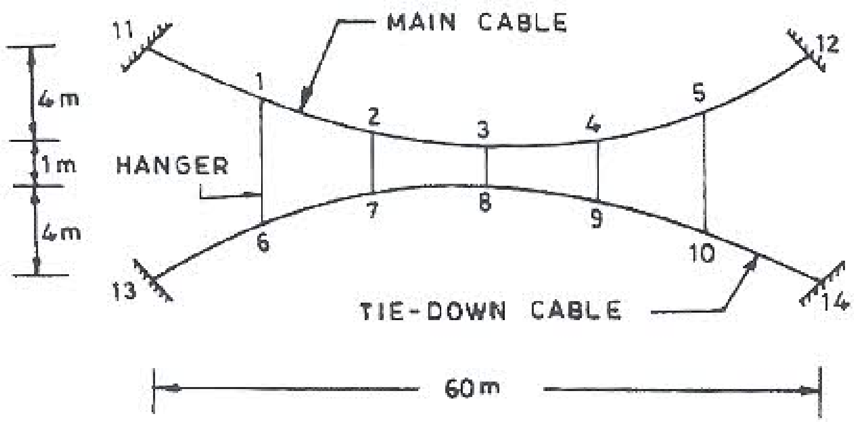

Figure 3 Cable truss studied (not to scale)

3.2 Loads

Rectangular and triangular type impulse loads are used for parametric study. Effects of symmetrical load, anti-symmetrical load, intensity of load, nature of load time step of integration on the response of the truss are studied as detailed below.

4. RESULTS AND DISCUSSION

4.1 Symmetrical load

A rectangular impulse of 200 N acting vertically downward with a duration of 0/08 sec. is applied at nodes 1 to 5. Node 3 undergoes the greatest displacement. Figure 4 gives the variation of the vertical displacement of node 3 with time. It is seen from the figure that both Wilson-θ Method and Newmark Method with β=1.4 give almost equal values. Also both linear and non-linear analyses give the same value since the cable truss studied is very stiff for the applied load. The same truss is later analysed for a rectangular impulse of 60 kN and a triangular impulse of 200 kN with a duration of 0.10 sec. each. Figures 5 and 6 give the displacement of node 1 under these loads, It can be seen from the figures that the difference in the linear and non-linear response values in the first cycle of vibration is about 7%.

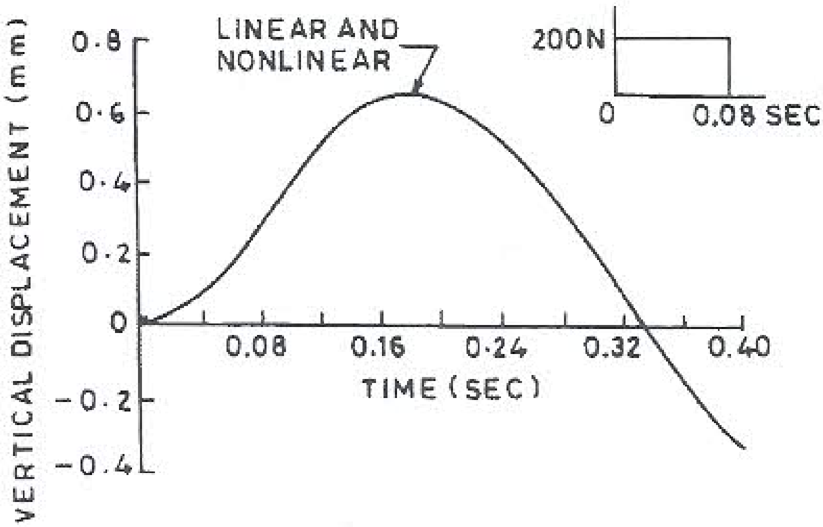

Figure 4 Displacement of node-3 under symmetrical load (By Wilson-θ method and Newmark method)

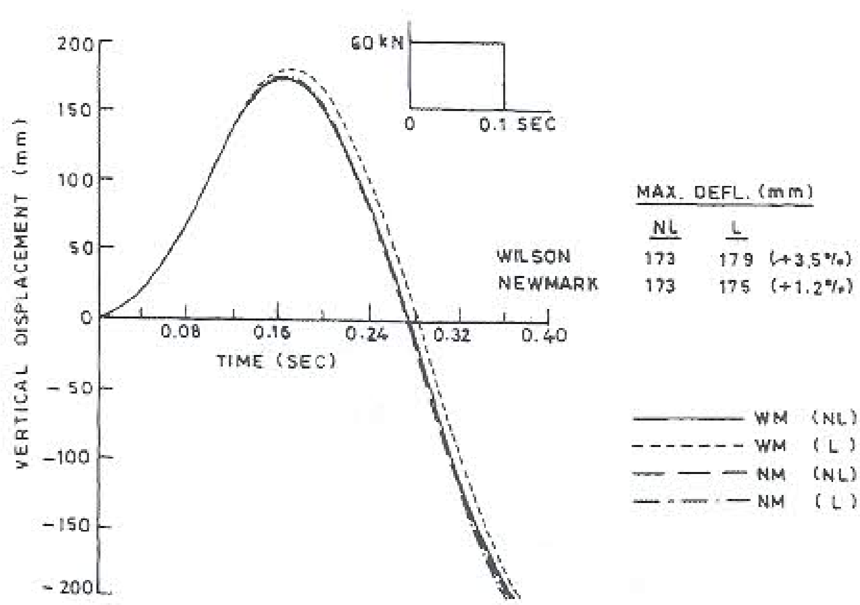

Figure 5 Displacement of node-1 under symmetrical load (rectangular impulse)

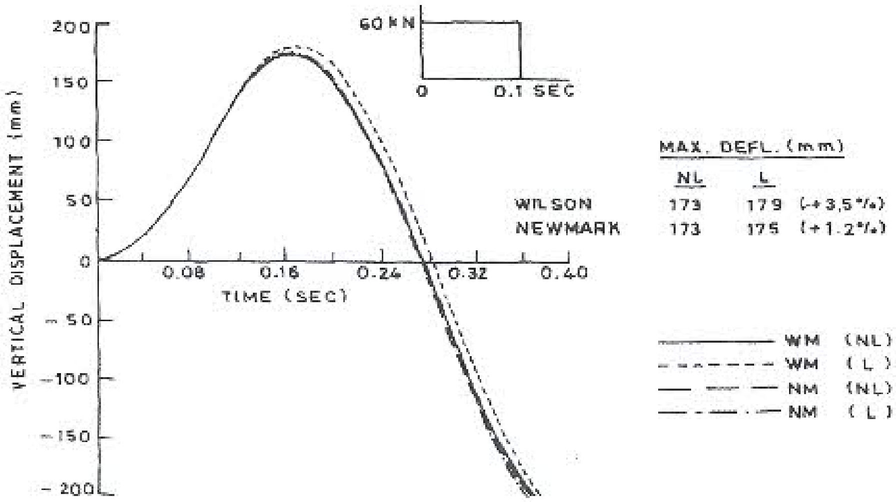

Figure 6 Displacement of node-1 under symmetrical load triangular impulse.

4.2 Anti-symmetrical load

A rectangular impulse of 200 N acting vertically downward is applied at nodes 1 and 2, and that acting vertically upward is applied at nodes 4 and 5 with a period of 0/08 sec. Node 2 undergoes the greatest displacement and node 3 the smallest. Displacement response of node 2 is shown in Figure 7. It is observed that in this case also both linear and non-linear solutions match. The same truss is later analysed for a rectangular impulse of 100 kN with a duration of 0/10 sec. applied anti-symmetrically on the upper cable. Figure 8 represents the response of node 2. It is seen from the figure that there is about 6/7% difference in the displacement values obtained from linear and non-linear analyses. Figure 9 shows the displacement of node 2 under a triangular impulse of 200 kN applied anti-symmetrically on the upper cable. A difference of about 8% is seen in the linear and non-linear response.

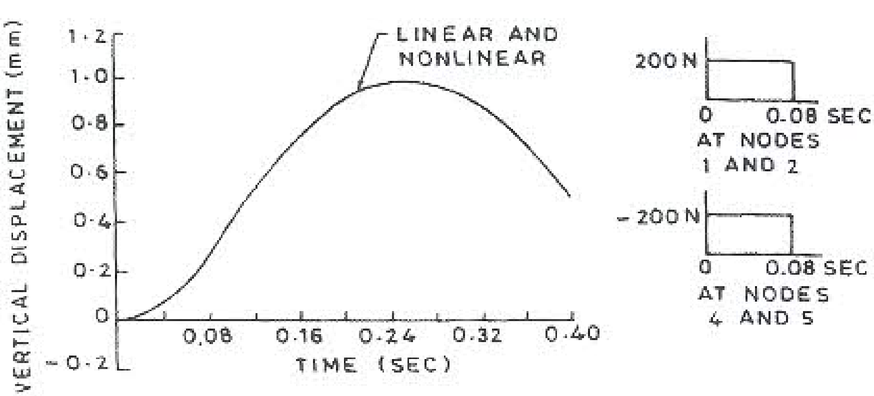

Figure 7 Displacement of node-2 under antisymmetrical load
(By Wilson-θ method and Newmark method)

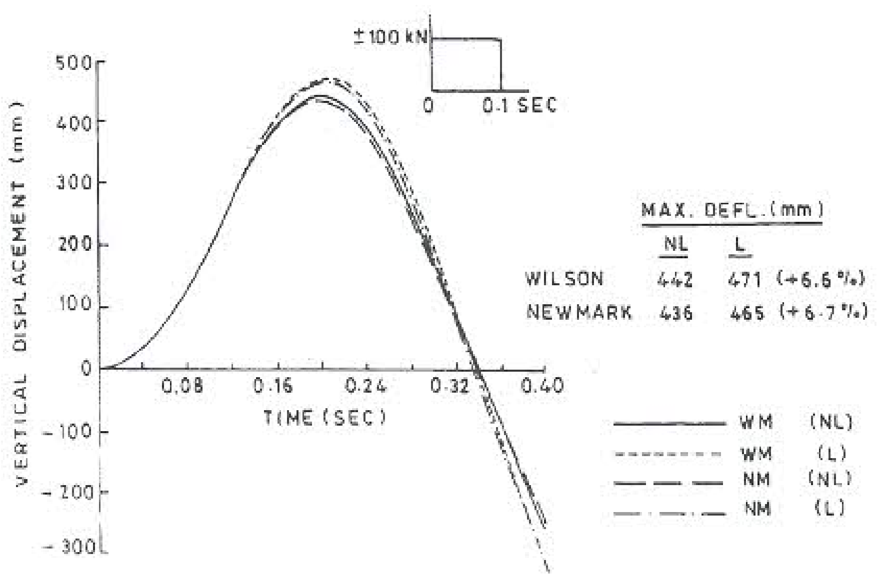

Figure 8 Displacement of node-2 under antisymmetrical load (rectangular impulse)

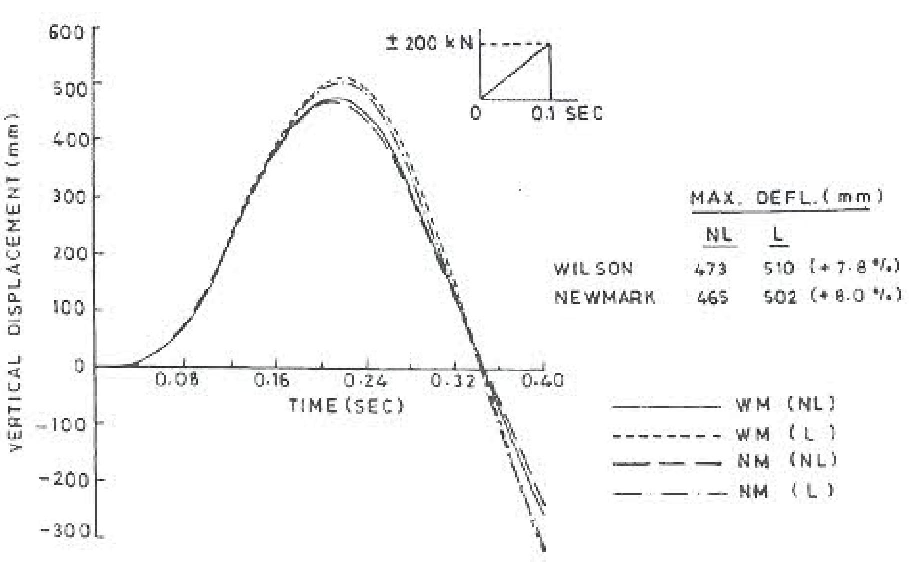

Figure 9 Displacement of node-2 under antisymmetrical load (triangular impulse)

4.3 Intensity of load

Displacement of node 3 due to three rectangular impulses with different amplitudes namely 200, 500 and 1000 N is shown in Figure. 10. It is observed that in all cases the equilibrium position is crossed at the same instance of time.

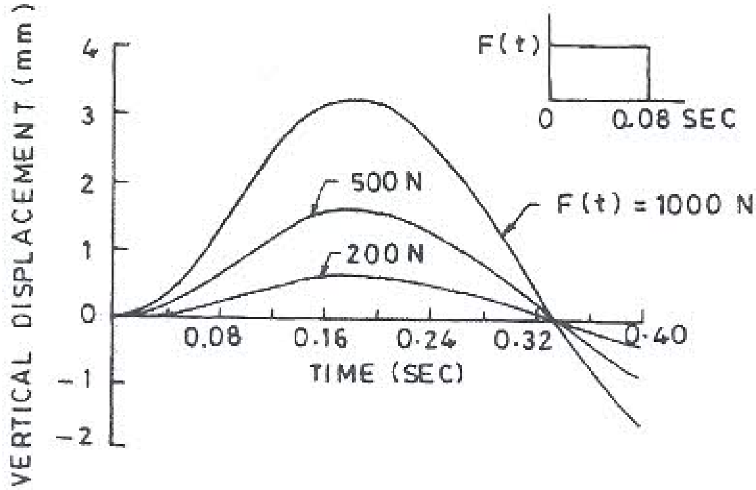

Figure 10 Effect of magnitude of the impulse load on displacement of node-3 (By Wilson-θ method and Newmark method)

4.4 Time step effect

Figure 11 shows the effect of time step of integration on displacement of node 3. The response of the truss is obtained under a rectangular impulse of 200 N with a duration of 0.10 sec. Three time steps namely 0.005, 0.010 and 0.020 sec. are taken. It is observed that although the results converge with all the three time steps (the solution techniques employed being unconditionally stable), larger time steps result in higher values of displacements, giving errors in the results. Further, with smaller time step number of iterations required for desired accuracy is reduced.

The effect of the time step of integration on the response of the truss is also studied under a rectangular impulse of 100 kN applied anti-symmetrically. Figure 12 and 13 give the variation of the vertical displacement of node 2 with time. Similar trends as mentioned above are observed. A time step of 0.020 sec. overestimates the results by about 5% compared to the results obtained with a time step of 0.005 sec.

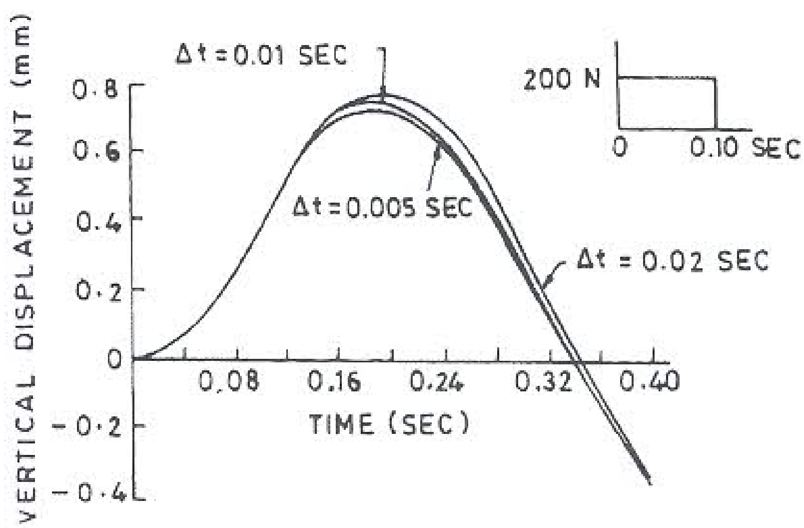

Figure 11 Effect of the time step of integration on displacement of node-3 (By Wilson-θ method and Newmark method)

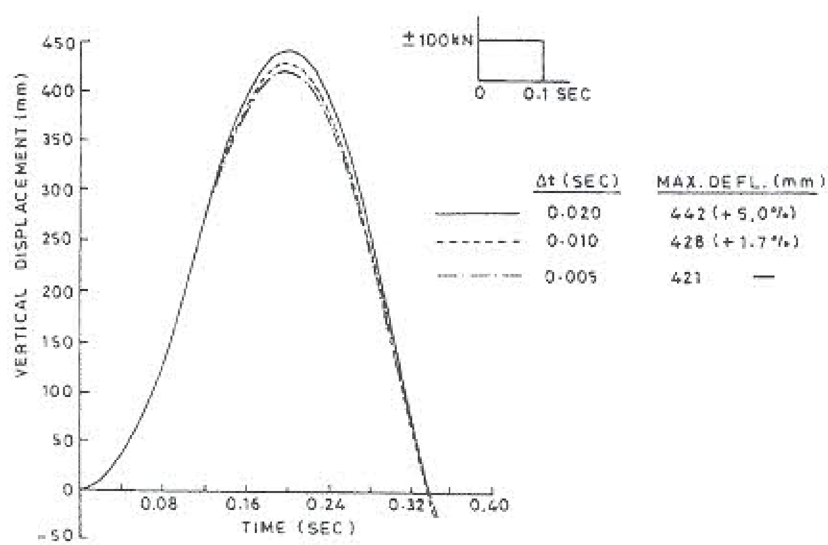

Figure 12 Effect of the time step of integration on displacement of node-2 (Wilson-θ method)

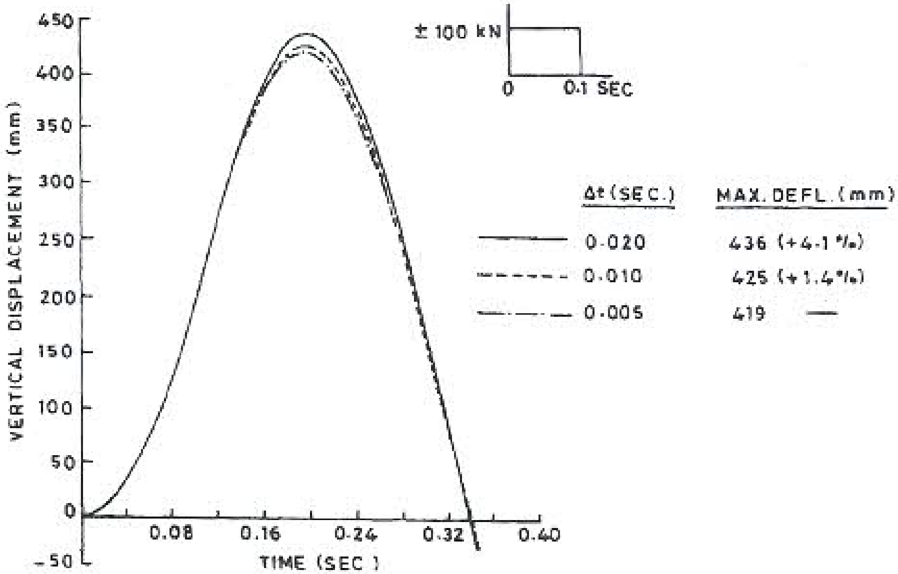

Figure 13 Effect of the time step of integration on displacement of node-2 (Newmark method)

4.5 Forcing function

Dynamic response of the truss for four different simple time functions is shown in Figure 14. It is seen from the figure that the impulse type-1 (rectangular) causes maximum deflections and impulse type-4 results in minimum deflections.

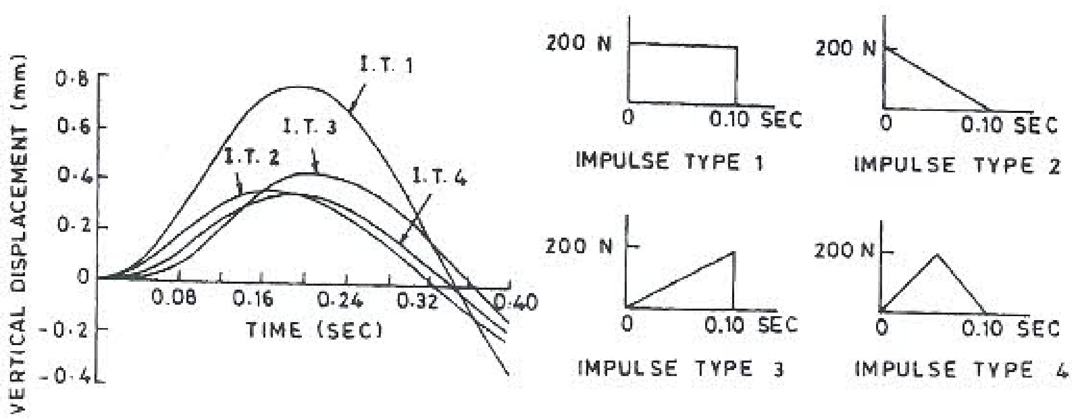

Figure 14 Effect of type of impulse on displacement of node-3
(By Wilson-θ method and Newmark method)

5. CONCLUSIONS

Following conclusions are drawn from the present study.

1. The dynamic response of the geometrically non-linear system such as pretensioned cable trusses can be successfully determined by dropping the non-linear terms from the equation of motion, but by updating the stiffness matrix after every iteration.
2. Non-linear analysis should be carried out while studying the response of cable trusses under dynamic loads. Linear analysis over-estimates the results. A cable truss show small degree of non-linearity under symmetrical loads compared to anti-symmetrical loads.

   Level of pretension in the truss also affects the difference in the values of linear and non-linear results. High degree of pretension will result in same or very close values of linear and non-linear response.
3. Maximum vertical displacement of the truss occurs at mid point of the truss under symmetrical loads. It occurs at a point between the centre and the support of the truss in case of anti-symmetrical loads.
4. The displacement of truss increases with the intensity of load. However, a point on the truss crosses the equilibrium position at the same instant of time for all the load intensities.
5. Larger time step of integration results in higher values of displacements, giving errors in the results.
6. The results of the proposed methods of integration for the solution of equations of dynamic equilibrium, namely Wilson-υ Method and Newmark Method with $\beta=1/4$ converge well and remain stable irrespective of the time step of integration.

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